M2 JUNE 08

1)

Diagram: incline with forces. "1600" up the slope, "14" down, box "2000kg", "$2000g \times \frac{1}{25}$", "$2000g \times \frac{1}{25} + \frac{P}{14}$"

$$\frac{P}{14} + \frac{2000g}{25} = 1600$$

$$\frac{P}{14} = 816 \implies P = 11424u$$

$$P = 11.4 \text{ kw} \quad (3sf)$$

2)

$$A \quad (4m) \xrightarrow{3u} \qquad B \quad (3m) \xrightarrow{2u} \qquad \text{before}$$

$$\xrightarrow{V_A} \qquad \qquad \xrightarrow{4eu} \quad \text{after}$$
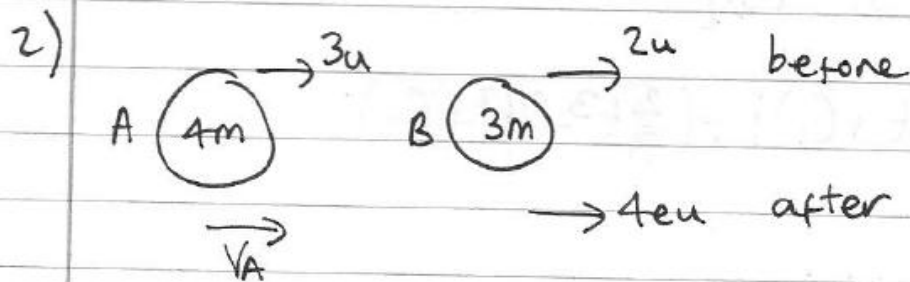
$$e = \frac{4eu - V_A}{u}$$

$$eu = 4eu - V_A$$

$$\implies V_A = 3eu.$$

$$\text{CLM} \implies 12mu + 6mu = 4m \times 3eu + 3m \times 4eu$$

$$18mu = 24meu \implies e = \frac{18}{24} \implies e = \frac{3}{4}$$

b)

$$KE_{before} = \tfrac{1}{2}(4m)(3u)^2 + \tfrac{1}{2}(3m)(2u)^2 = 24mu^2$$

$$V_A = 3eu = 3\left(\tfrac{3}{4}\right)u = \tfrac{9}{4}u \qquad V_B = 4eu = 4\left(\tfrac{3}{4}\right)u = 3u$$

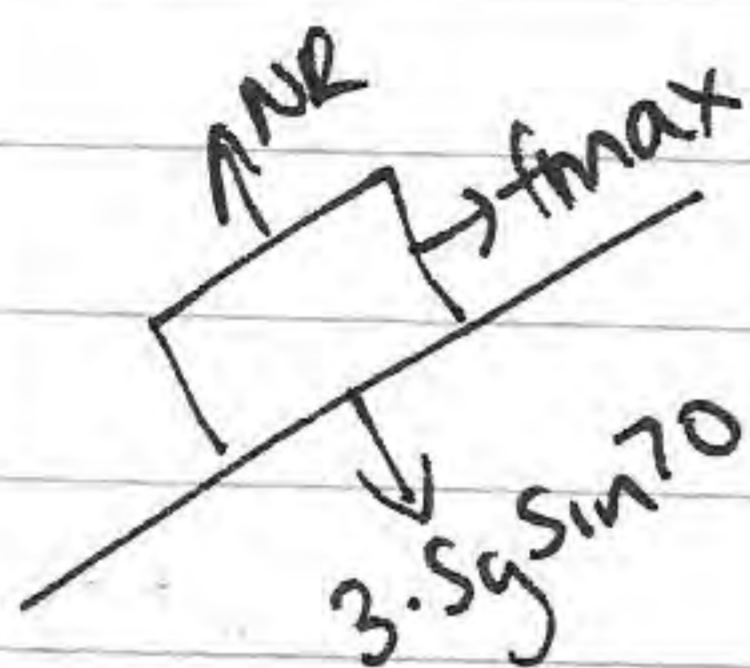$$KE_{after} = \tfrac{1}{2}(4m)\left(\tfrac{9}{4}u\right)^2 + \tfrac{1}{2}(3m)(3u)^2 = \tfrac{189}{8}mu^2$$

$$\implies KE_{lost} = \tfrac{3}{8}mu^2 \text{ J}.$$

3) loss in KE $= \frac{1}{2}(3.5)(12^2 - 8^2) = 140_J$

loss in PE $= 3.5g(14\sin 20) = 164.238\ldots_J$

Total Energy lost $= \underline{304_J}$ (3sf)

b) Total Energy lost = Wd against friction $= f_{max} \times 14$

$f_{max} = \mu N_R = \mu(3.5g \sin 70)$

$304.238\ldots = \mu \times 32.2314\ldots \times 14$

$\mu = \underline{0.67}$ (2sf)

4) $F = (6t-5)i + (t^2-2t)j = ma = \frac{1}{2}a$

$\Rightarrow a = (12t-10)i + (2t^2-4t)j$

$v = \int a\, dt = (6t^2 - 10t + C_1)i + (\frac{2}{3}t^3 - 2t^2 + C_2)j$

$t=0 \quad v = i - 4j = C_1 i + C_2 j \qquad C_1 = 1 \quad C_2 = -4$

$v = (6t^2 - 10t + 1)i + (\frac{2}{3}t^3 - 2t^2 - 4)j$

$t=3, \quad v = 25i + -4j$

Mom at $t=3 = mv = 12.5i - 2j$

$+$ Impulse $\qquad = -5i + 12j$

$\Rightarrow$ Mom after $= mv = 7.5i + 10j$

$\Rightarrow v$ after $= 15i + 20j$

$\Rightarrow$ Speed $= \sqrt{15^2 + 20^2} = \underline{25\ ms^{-1}}$

5)
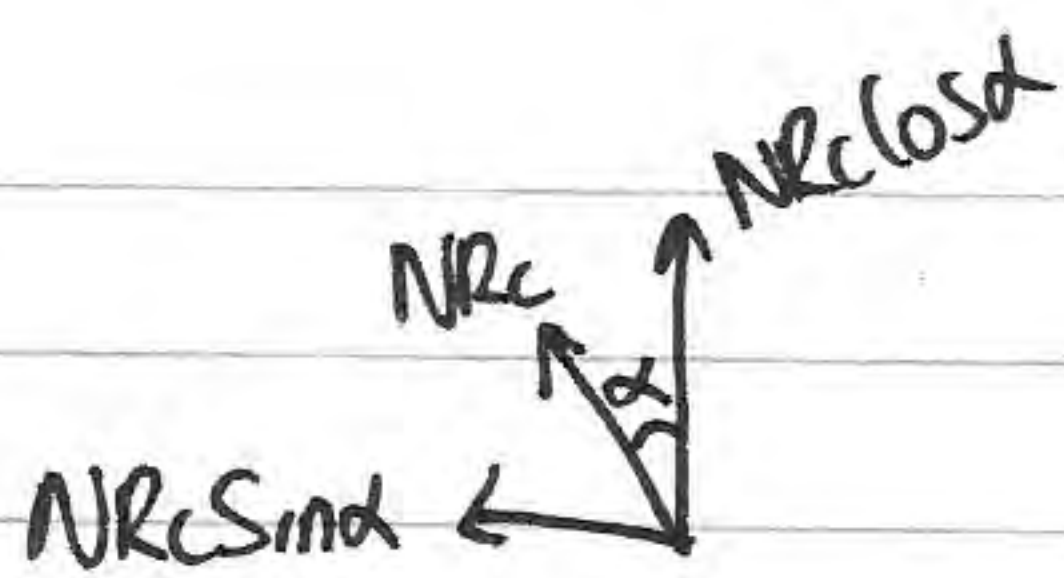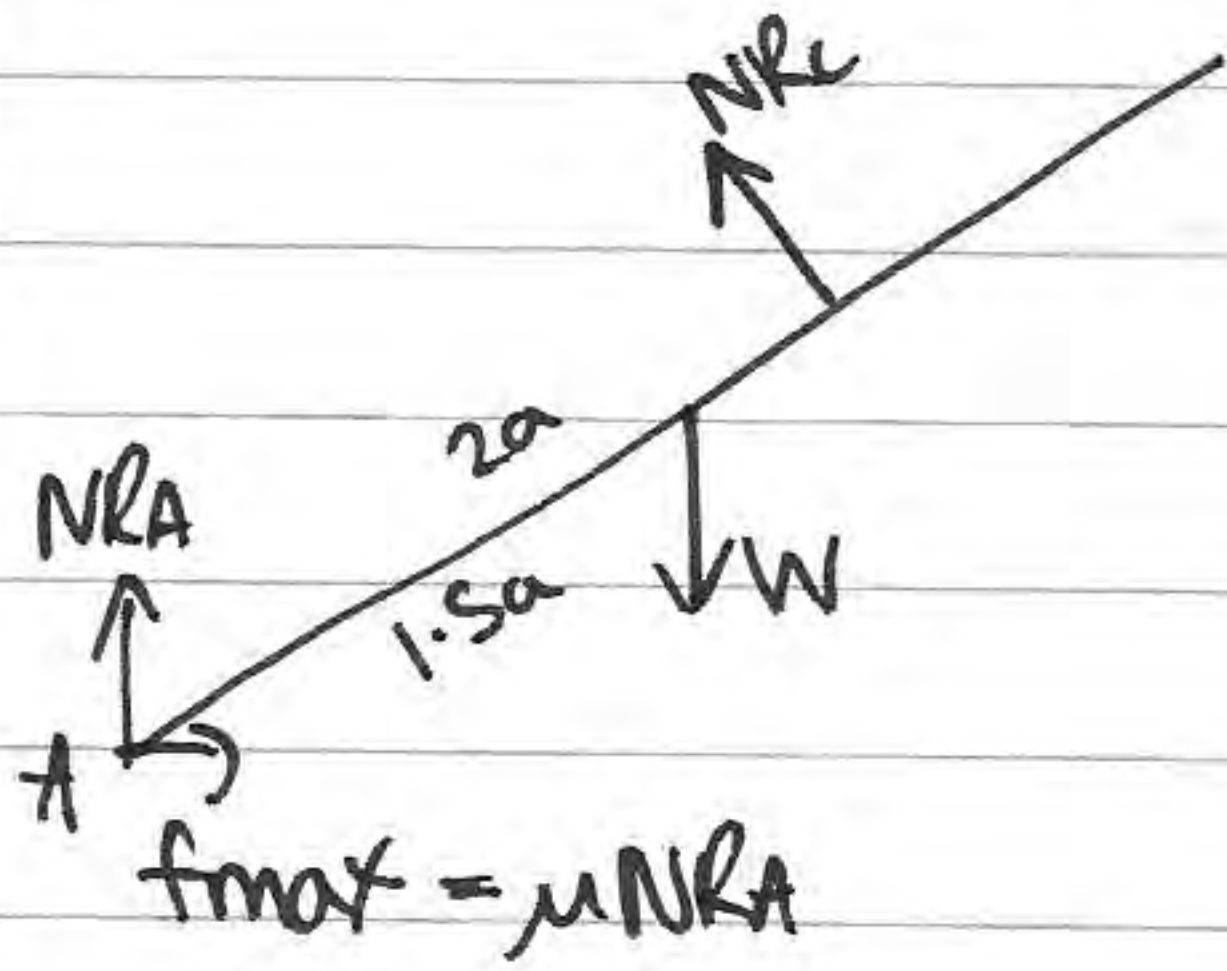
$Rf \uparrow = 0$

$NRc \cos\alpha + NRA = W$

$\overrightarrow{Rf} = 0$
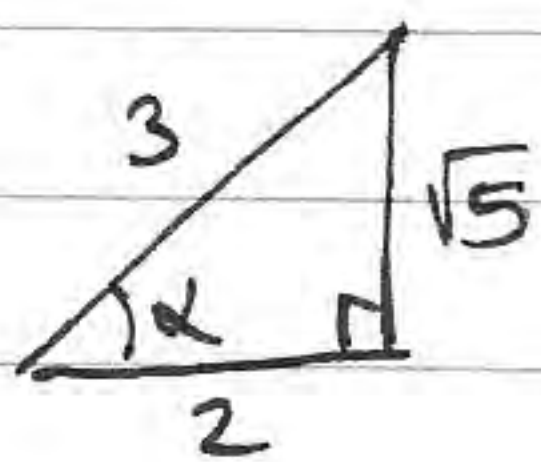
$NRc \sin\alpha = fmax = \mu NRA$

$A \circlearrowright \quad W \times 1.5a \cos\alpha = NRc \times 2a \implies NRc = \frac{3}{4} W \cos\alpha$

$\implies \left(\frac{3}{4} W \cos\alpha\right) \cos\alpha + NRA = W \implies NRA = W - \frac{3}{4} W \cos^2\alpha$

$\therefore NRA = \frac{1}{4} W (4 - 3\cos^2\alpha) \text{ \#}$
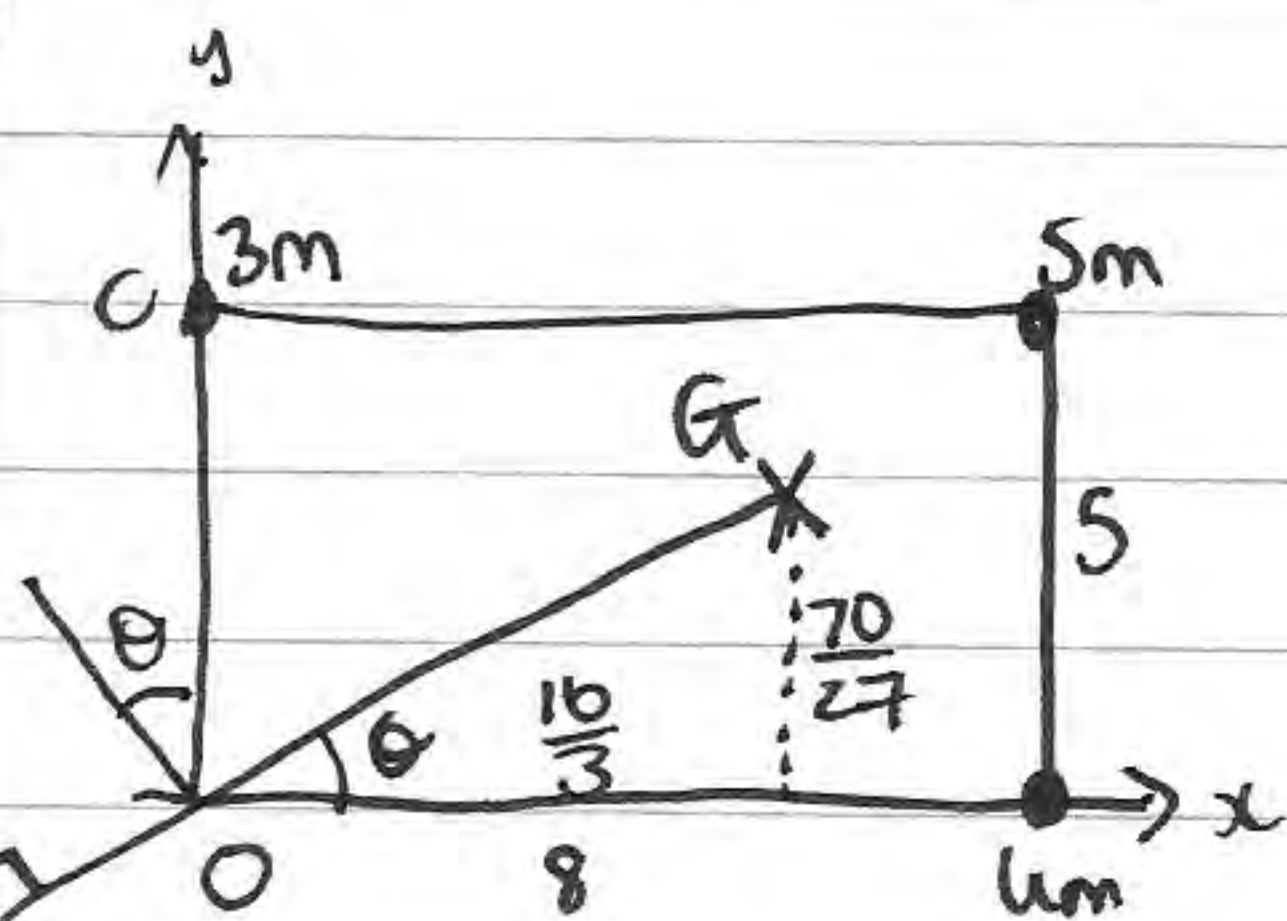
$\cos\alpha = \frac{2}{3}$

$\sin\alpha = \frac{\sqrt{5}}{3}$

$NRA = \frac{1}{4} W \left(4 - \left(\frac{2}{3}\right)^2\right) = \frac{2}{3} W$

$NRc = \frac{3}{4} W \left(\frac{2}{3}\right) = \frac{1}{2} W$

$\implies \left(\frac{1}{2} W\right) \frac{\sqrt{5}}{3} = \mu \left(\frac{2}{3} W\right) \implies \frac{\sqrt{5}}{6} = \frac{2}{3} \mu \implies \mu = \frac{\sqrt{5}}{4}$

6

$\uparrow \quad 3mg \times 0 + 5mg \times 8 + 4mg \times 8 = (8+4)mg \times \bar{x}$

$\implies 40 + 8\mu = (8+\mu) \times 6.4$

$\implies 1.6\mu = 11.2$

$\implies \mu = 7 \text{ \#}$

c) $\theta = \tan^{-1}\left(\frac{\frac{70}{27}}{\frac{16}{3}}\right)$

$\theta = \tan^{-1}\left(\frac{35}{72}\right)$

$\theta = 25.9° (3sf)$

$\uparrow y \quad 15mg \times 6.4 + 12mg \times 4 = 27mg \times \bar{x}$

$\bar{x} = \frac{16}{3}$
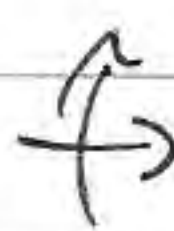
$\uparrow x \quad 7mg \times 0 + 5mg \times 5 + 3mg \times 5 + 12mg \times 2.5 = 27mg \bar{y}$

$\implies 70 = 27\bar{y} \implies y = \frac{70}{27} \quad G\left(\frac{16}{3}, \frac{70}{27}\right)$

7) $(v\downarrow)$ $u\downarrow = 25\sin30 = 12.5$ $s = ut + \frac{1}{2}at^2$

$a\downarrow = 9.8$ $12 = 12.5t + 4.9t^2$

$s\downarrow = 12$ $4.9t^2 + 12.5t - 12 = 0$

$t = 0.743377...$

$(\overrightarrow{H})$ $Vel = 25\cos30 = \dfrac{25\sqrt{3}}{2}$ $x = OB$ $t = 0.743377...$

$t = 0.74 \ (2sf)$

b) $OB = \dfrac{25\sqrt{3}}{2} \times 0.743377... = 16.1$ $\Rightarrow$ $TB = \underline{1.1\text{m} \ (2sf)}$

c) $(\overrightarrow{H})$ $x = 15$ $Vel = \dfrac{25\sqrt{3}}{2}$ $\Rightarrow$ $t = \dfrac{15}{\frac{25\sqrt{3}}{2}} = \dfrac{6}{5\sqrt{3}}$

$(v\downarrow)$ $u\downarrow = 12.5$ $v\downarrow = u + at = 12.5 + 9.8 \times \dfrac{6}{5\sqrt{3}}$

$a\downarrow = 9.8$

$t = \dfrac{6}{5\sqrt{3}}$ $v\downarrow = 19.289639...$

$\dfrac{25\sqrt{3}}{2}$

Speed $\downarrow 19.289639...$ $\Rightarrow$ $Speed = \sqrt{19.289639...^2 + \left(\dfrac{25\sqrt{3}}{2}\right)}$

$Speed = 29 \text{ ms}^{-1} \ (2sf)$